**SC531 PROBABILITY & RANDOM VARIABLES**

**FINAL EXAMINATION**

Each question carries 3 marks. Time allowed: 80 minutes.

Use the values of M & N you have calculated earlier.

Q-1. You have 5 bowls containing, respectively, 10, 10, 20, 20 and 20 balls. Some balls in each bowl are white; the rest are black. The number of white balls in the bowls are, respectively, M, 10-M, 11, 4 and 15. A bowl is selected at random, and a single ball is drawn from that bowl at random. It is found that the ball drawn is **black**. Find the probability that the drawn **black** ball came from bowl #2.  
  
Q-2. A fair coin is tossed 10 times. Find the probability that it turns up HEAD either N-1 or N times.

pdf of continuous RV X (for Q-3)

X = 0 X = 8 X = 20 X axis 🡪

Q-3. Given the probability density function shown in the diagram above, find Prob[ M <= X <= 2\*N ].

Q-4. At a router, the number of packets arriving per millisecond, denoted by P, follows Poisson distribution, with mean rate of 2 packets per millisecond. Find Prob[ M < P < M+3 ]. Refer to the table below.

|  |  |  |
| --- | --- | --- |
| **Poisson distribution, rate = 2** |  |  |
| **count, x** | **p(x)** |  |
| 0 | 0.1353 |  |
| 1 | 0.2707 |  |
| 2 | 0.2707 |  |
| 3 | 0.1804 |  |
| 4 | 0.0902 |  |
| 5 | 0.0361 |  |
| 6 | 0.0120 |  |
| 7 | 0.0034 |  |
| 8 | 0.0009 |  |
| 9 | 0.0002 |  |
| 10 | 0.0000 |  |

Q-5. The average working life of a certain power supply is claimed to be 10000 hours, with standard deviation of 400\*M hours. We test a sample of size 25 of the power supplies, and calculate the sample mean. Find the probability that the sample mean is between 9600 and 10400 hours.

Note: For counts > 10, the probability values are negligible.

**Standard normal distribution -- cumulative**

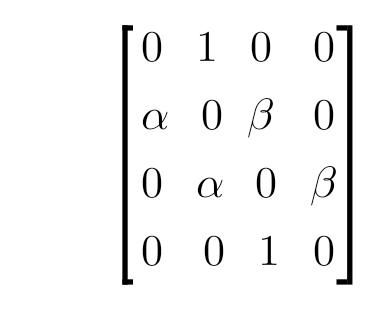


|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Index | z | F(z) | Index | z | F(z) |
| 1 | 0.00 | 0.5000 | 11 | 1.00 | 0.8413 |
| 2 | 0.10 | 0.5398 | 12 | 1.10 | 0.8643 |
| 3 | 0.20 | 0.5793 | 13 | 1.20 | 0.8849 |
| 4 | 0.30 | 0.6179 | 14 | 1.30 | 0.9032 |
| 5 | 0.40 | 0.6554 | 15 | 1.40 | 0.9192 |
| 6 | 0.50 | 0.6915 | 16 | 1.50 | 0.9332 |
| 7 | 0.60 | 0.7257 | 17 | 1.60 | 0.9452 |
| 8 | 0.70 | 0.7580 | 18 | 1.70 | 0.9554 |
| 9 | 0.80 | 0.7881 | 19 | 1.80 | 0.9641 |
| 10 | 0.90 | 0.8159 | 20 | 1.90 | 0.9713 |
|  |  |  | 21 | 2.00 | 0.9772 |

Q-6. Five pairs of values of random variables X and Y are tabulated below. Find the COVARIANCE of X & Y.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 1 | 2 | 3 | 4 | 5 |
| Y | -3M | -4M | 0 | 3M | 4M |

Q-7. Trucks arrive at a toll booth at the average rate of 12 arrivals per hour, and the arrivals define a Poisson process. What is the probability that the time interval T between two successive arrivals, measured in minutes, satisfies M < T < N?  
  
Q-8. Recall the Markov process defined as "random walk with reflecting barriers". The four states of the process are 1, 2, 3 and 4. The transition probability matrix is as given below, with a = M/10. The initial probability distribution over states is (1/4, 1/4, 1/4, 1/4). What is the probability that the process is in state 1 after two time steps?

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